Page 258, equations (4.508)-(4.513)

It follows from (4.137):

$$\sum_{\delta=1}^{n-1} \nu_{\beta\delta} \mathbf{u}_\delta = -\mathbf{k}_\beta - \xi_\beta \mathbf{g} + \sum_{\gamma=1}^{n} \omega_{\beta\gamma} \mathbf{h}_\gamma,\ \beta = 1, ..., n-1, \tag{1}$$

from (4.58):

$$\mathbf{k}_\alpha = \rho_\alpha \dot{\mathbf{v}}_\alpha - \operatorname{div} \mathbf{T}_\alpha - \rho_\alpha (\mathbf{b}_\alpha + \mathbf{i}_\alpha),\ \alpha = 1, ..., n, \tag{2}$$

and from (4.505):

$$\operatorname{div} \mathbf{T}_\alpha = -\text{grad} P_\alpha + \operatorname{div} \mathbf{T}_\alpha^N,\ \alpha = 1, ..., n. \tag{3}$$

Substituting from (2) and (3) into (1) we obtain:

$$\sum_{\delta=1}^{n-1} \nu_{\beta\delta} \mathbf{u}_\delta = -\rho_\beta \dot{\mathbf{v}}_\beta - \text{grad} P_\beta + \operatorname{div} \mathbf{T}_\beta^N + \rho_\beta (\mathbf{b}_\beta + \mathbf{i}_\beta) - \xi_\beta \mathbf{g} + \sum_{\gamma=1}^{n} \omega_{\beta\gamma} \mathbf{h}_\gamma$$

which is (4.508).

Gradient form of (4.208) can be written using (4.123) and (3.112) or (4.124) as

$$\sum_{\gamma=1}^{n} \omega_{\beta\gamma} \mathbf{h}_\gamma = \text{grad} P_\beta - \rho_\beta\, \text{grad} g_\beta + \rho_\beta \frac{\partial \hat{f}_\beta}{\partial T} \mathbf{g},\ \beta = 1, ..., n-h. \tag{4}$$

Adding to the right hand side of (4) "zero" expression $\rho_\beta s_\beta \mathbf{g} - \rho_\beta s_\beta \mathbf{g}$ and using the definition (4.510) we get

$$\sum_{\gamma=1}^{n} \omega_{\beta\gamma} \mathbf{h}_\gamma = \text{grad} P_\beta - \rho_\beta\, \text{grad}_T g_\beta + \rho_\beta (s_\beta + \frac{\partial \hat{f}_\beta}{\partial T}) \mathbf{g},\ \beta = 1, ..., n-h$$

which is (4.509).

Capitalizing upon (4.216) the gradient of chemical potential can be written as

$$\begin{aligned} \text{grad} g_\alpha &= \frac{\partial \tilde{g}_\alpha}{\partial T} \text{grad} T + \frac{\partial \tilde{g}_\alpha}{\partial P} \text{grad} P + \sum_{\beta=1}^{n-1} \frac{\partial \tilde{g}_\alpha}{\partial w_\beta} \text{grad} w_\beta \\ &= -s_\alpha \mathbf{g} + v_\alpha\, \text{grad} P + \sum_{\beta=1}^{n-1} \frac{\partial \tilde{g}_\alpha}{\partial w_\beta} \text{grad} w_\beta \end{aligned} \tag{5}$$

where (4.266) and (4.267) was used in the last equality together with the designation (3.112) or (4.124). Substituting (5) into (4.510), eq.(4.511) follows immediately.

Introduction of (4.509) into (4.508) gives:

$$\sum_{\delta=1}^{n-1} \nu_{\beta\delta}\mathbf{u}_\delta = -\rho_\beta \,\mathrm{grad}_T g_\beta + \rho_\beta (s_\beta + \frac{\partial \hat{f}_\beta}{\partial T})\mathbf{g} - \xi_\beta \mathbf{g} + \mathrm{div}\,\mathbf{T}_\beta^N + \rho_\beta(\mathbf{b}_\beta + \mathbf{i}_\beta)$$

$$- \rho_\beta \grave{\mathbf{v}}_\beta = -\rho_\beta \mathbf{y}_\beta + \rho_\beta (s_\beta + \frac{\partial \hat{f}_\beta}{\partial T})\mathbf{g} - \xi_\beta \mathbf{g} \quad (6)$$

(the definition (4.512) was used). It follows from (4.163):

$$\rho_\beta s_\beta + \rho_\beta \frac{\partial \hat{f}_\beta}{\partial T} - \xi_\beta = \frac{\lambda_\beta}{T} - \vartheta_\beta,\ \beta = 1, ..., n-h. \quad (7)$$

Substituting from (7) to (6), eq.(4.513) follows.