Page 166, equation (4.87)

It follows from (4.82):

$$-\mathrm{div}\mathbf{q} + Q = \sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha u_\alpha}{\partial t} + \sum_{\alpha=1}^{n} \mathrm{div}(\rho_\alpha u_\alpha \mathbf{v}_\alpha) - \sum_{\alpha=1}^{n} \mathrm{tr}\, \mathbf{T}_\alpha \mathbf{D}_\alpha + \sum_{\beta=1}^{n-1} \mathbf{k}_\beta . \mathbf{u}_\beta + (1/2) \sum_{\beta=1}^{n-1} r_\beta \mathbf{u}_\beta^2. \quad (1)$$

The divergence in (4.84) can be expressed (summation convention employed):

$$\mathrm{div}\,(\mathbf{q}/T) \equiv \frac{\partial}{\partial x^i}(q^i/T) = (1/T)\frac{\partial q^i}{\partial x^i} + q^i \frac{\partial}{\partial x^i}(1/T) = (1/T)\frac{\partial q^i}{\partial x^i} - (q^i/T^2)\frac{\partial T}{\partial x^i} \equiv (1/T)\mathrm{div}\,\mathbf{q} - (1/T^2)\mathbf{q}.\mathbf{g}. \quad (2)$$

Substituting (1) into (4.84) multiplied by $-T$ and taking into account (2) we obtain:

$$-T\sigma \equiv -T\sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha s_\alpha}{\partial t} - T\sum_{\alpha=1}^{n} \mathrm{div}(\rho_\alpha s_\alpha \mathbf{v}_\alpha) + (1/T)\mathbf{q}.\mathbf{g} + \sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha u_\alpha}{\partial t} + \sum_{\alpha=1}^{n} \mathrm{div}(\rho_\alpha u_\alpha \mathbf{v}_\alpha) - \sum_{\alpha=1}^{n} \mathrm{tr}\, \mathbf{T}_\alpha \mathbf{D}_\alpha + \sum_{\beta=1}^{n-1} \mathbf{k}_\beta . \mathbf{u}_\beta + (1/2) \sum_{\beta=1}^{n-1} r_\beta \mathbf{u}_\beta^2. \quad (3)$$

From definition (4.86) $\rho_\alpha f_\alpha = \rho_\alpha u_\alpha - T\rho_\alpha s_\alpha$ follows, further,

$$\frac{\partial (T\rho_\alpha s_\alpha)}{\partial t} = \rho_\alpha s_\alpha \frac{\partial T}{\partial t} + T\frac{\partial \rho_\alpha s_\alpha}{\partial t};$$

thus following relationship is valid

$$-T\sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha s_\alpha}{\partial t} = \sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha f_\alpha}{\partial t} - \sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha u_\alpha}{\partial t} + \sum_{\alpha=1}^{n} \rho_\alpha s_\alpha \frac{\partial T}{\partial t}. \quad (4)$$

Similarly

$$\mathrm{div}(T\rho_\alpha s_\alpha \mathbf{v}_\alpha) = \rho_\alpha s_\alpha \mathbf{v}_\alpha . \mathbf{g} + T\mathrm{div}(\rho_\alpha s_\alpha \mathbf{v}_\alpha) = \mathrm{div}(\rho_\alpha u_\alpha \mathbf{v}_\alpha) - \mathrm{div}(\rho_\alpha f_\alpha \mathbf{v}_\alpha)$$

and

$$-T\sum_{\alpha=1}^{n} \mathrm{div}(\rho_\alpha s_\alpha \mathbf{v}_\alpha) = \sum_{\alpha=1}^{n} \rho_\alpha s_\alpha \mathbf{v}_\alpha . \mathbf{g} - \sum_{\alpha=1}^{n} \mathrm{div}(\rho_\alpha u_\alpha \mathbf{v}_\alpha) + \sum_{\alpha=1}^{n} \mathrm{div}(\rho_\alpha f_\alpha \mathbf{v}_\alpha) \quad (5)$$

Further, $\mathrm{div}(\rho_\alpha f_\alpha \mathbf{v}_\alpha) = \rho_\alpha f_\alpha \mathrm{div}\, \mathbf{v}_\alpha + \mathbf{v}_\alpha . \mathrm{grad}(\rho_\alpha f_\alpha)$ and substituting this relationship, (4), and (5) into (3), eq.(4.87) follows.