

Page 236, equation (4.427)

Making partial derivative of definition (4.86) gives:

$$\frac{\partial \hat{f}_\alpha}{\partial \rho_\alpha} = \frac{\partial \hat{u}_\alpha}{\partial \rho_\alpha} - T \frac{\partial \hat{s}_\alpha}{\partial \rho_\alpha}, \quad \alpha = 1, \dots, n. \quad (1)$$

From (4.426) we get:

$$\begin{aligned} \frac{\partial \hat{s}_\alpha}{\partial \rho_\alpha} &= -\frac{\partial}{\partial \rho_\alpha} \left(\frac{\partial \hat{f}_\alpha}{\partial T} \right) = -\frac{\partial}{\partial T} \left(\frac{\partial \hat{f}_\alpha}{\partial \rho_\alpha} \right) = -\frac{\partial}{\partial T} \left(\frac{RT}{M_\alpha \rho_\alpha} \right) = \\ &= -\frac{R}{M_\alpha \rho_\alpha}, \quad \alpha = 1, \dots, n \end{aligned} \quad (2)$$

where (4.429) was used in the last but one equality. Substituting (4.429) and (2) into (1) gives:

$$\frac{RT}{M_\alpha \rho_\alpha} = \frac{\partial \hat{u}_\alpha}{\partial \rho_\alpha} + \frac{RT}{M_\alpha \rho_\alpha} \quad \Rightarrow \quad \frac{\partial \hat{u}_\alpha}{\partial \rho_\alpha} = 0, \quad \alpha = 1, \dots, n.$$

Consequently, $\hat{u}_\alpha(T, \rho_\gamma)$ simplifies to $\hat{u}_\alpha(T)$ – cf. also (4.86), (4.413), and (4.426).