Page 168, equation (4.101)

Let us denote the last two terms of (4.82) by L and modify them using (4.24):

$$\begin{aligned}
L &= -\sum_{\beta=1}^{n-1} \mathbf{k}_\beta.\mathbf{u}_\beta - (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{u}_\beta^2 \\
&= -\sum_{\beta=1}^{n-1} \mathbf{k}_\beta.(\mathbf{v}_\beta - \mathbf{v}_n) - (1/2)\sum_{\beta=1}^{n-1} r_\beta (\mathbf{v}_\beta - \mathbf{v}_n)^2 \\
&= -\sum_{\beta=1}^{n-1} \mathbf{k}_\beta.\mathbf{v}_\beta + \sum_{\beta=1}^{n-1} \mathbf{k}_\beta.\mathbf{v}_n - (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta^2 + \sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta.\mathbf{v}_n - (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_n^2.
\end{aligned} \tag{1}$$

From (4.98) we obtain:

$$\begin{aligned}
\sum_{\alpha=1}^{n} \mathbf{k}_\alpha &= -\sum_{\alpha=1}^{n} r_\alpha \mathbf{v}_\alpha, \\
\sum_{\alpha=1}^{n} \mathbf{k}_\alpha.\mathbf{v}_n &= -\sum_{\alpha=1}^{n} r_\alpha \mathbf{v}_\alpha.\mathbf{v}_n, \\
\sum_{\beta=1}^{n-1} \mathbf{k}_\beta.\mathbf{v}_n &= -\mathbf{k}_n.\mathbf{v}_n - \sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta.\mathbf{v}_n - r_n \mathbf{v}_n^2.
\end{aligned} \tag{2}$$

Substituting from (2) into (1):

$$\begin{aligned}
L = &-\sum_{\beta=1}^{n-1} \mathbf{k}_\beta.\mathbf{v}_\beta - \mathbf{k}_n.\mathbf{v}_n - \sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta.\mathbf{v}_n - r_n \mathbf{v}_n^2 - (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta^2 \\
&+ \sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta.\mathbf{v}_n - (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_n^2 = -\sum_{\alpha=1}^{n} \mathbf{k}_\alpha.\mathbf{v}_\alpha - r_n \mathbf{v}_n^2 \\
&- (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta^2 - (1/2)\sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_n^2.
\end{aligned} \tag{3}$$

We have from (4.96) $\sum_{\beta=1}^{n-1} r_\beta = -r_n$ and substituting this into (3):

$$\begin{aligned} L &= -\sum_{\alpha=1}^{n} \mathbf{k}_\alpha . \mathbf{v}_\alpha - r_n \mathbf{v}_n^2 - (1/2) \sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta^2 - (1/2)(-r_n \mathbf{v}_n^2) \\ &= -\sum_{\alpha=1}^{n} \mathbf{k}_\alpha . \mathbf{v}_\alpha - (1/2) r_n \mathbf{v}_n^2 - (1/2) \sum_{\beta=1}^{n-1} r_\beta \mathbf{v}_\beta^2 \\ &= -\sum_{\alpha=1}^{n} \mathbf{k}_\alpha . \mathbf{v}_\alpha - (1/2) \sum_{\alpha=1}^{n} r_\alpha \mathbf{v}_\alpha^2 . \end{aligned} \tag{4}$$

Substituting from (3) into (4.82), eq. (4.101) follows.