Page 87, equations (3.67) and (3.68)

The proof of equation (3.67):

The left hand side can be expanded

$$\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\rho\varphi\mathbf{v} = \varphi\dot{\rho} + \rho\dot{\varphi} + \varphi\mathrm{div}\rho\mathbf{v} = \varphi\left(\frac{\partial \rho}{\partial t} + \mathrm{div}\rho\mathbf{v}\right) + \rho\dot{\varphi}$$

Using (3.62), equation (3.67) follows.

The proof of equation (3.68):

From equation (3.22) it follows:

$$\begin{aligned}\dot{\int_{\mathcal{V}} \rho\varphi \,\mathrm{d}v} &= \int_{\mathcal{V}} \left(\dot{\overline{\rho\varphi}} + \rho\varphi \,\mathrm{div}\mathbf{v}\right) \mathrm{d}v = \int_{\mathcal{V}} \left(\varphi\dot{\rho} + \rho\dot{\varphi} + \rho\varphi \,\mathrm{div}\mathbf{v}\right) \mathrm{d}v \\ &= \int_{\mathcal{V}} \left[\varphi\left(\frac{\partial \rho}{\partial t} + \mathrm{div}\rho\mathbf{v}\right) + \rho\dot{\varphi}\right] \mathrm{d}v\end{aligned}$$

Using (3.62), equation (3.68) follows.