Page 120, equation (3.209)

Entropy derivatives follow from (3.208) as:

$$\frac{\partial \tilde{s}}{\partial T} = (1/T)\left(\frac{\partial \tilde{u}}{\partial T} - (P/\rho^2)\frac{\partial \tilde{\rho}}{\partial T}\right), \quad (1)$$

$$\frac{\partial \tilde{s}}{\partial P} = (1/T)\left(\frac{\partial \tilde{u}}{\partial P} - (P/\rho^2)\frac{\partial \tilde{\rho}}{\partial P}\right). \quad (2)$$

From (1) and (2) we obtain for the second derivatives:

$$\frac{\partial^2 \tilde{s}}{\partial T \partial P} = (1/T)\left(\frac{\partial^2 \tilde{u}}{\partial T \partial P} - (1/\rho^2)\frac{\partial \tilde{\rho}}{\partial T} - (P/\rho^2)\frac{\partial^2 \tilde{\rho}}{\partial T \partial P}\right) \quad (3)$$

and

$$\begin{aligned}\frac{\partial^2 \tilde{s}}{\partial P \partial T} &= -(1/T^2)\left(\frac{\partial \tilde{u}}{\partial P} - (P/\rho^2)\frac{\partial \tilde{\rho}}{\partial P}\right) \\ &\quad + (1/T)\left(\frac{\partial^2 \tilde{u}}{\partial T \partial P} - (P/\rho^2)\frac{\partial^2 \tilde{\rho}}{\partial P \partial T}\right). \end{aligned} \quad (4)$$

The derivatives in (3) and (4) should be equal, hence:

$$\begin{aligned} -(1/T)(1/\rho^2)\frac{\partial \rho}{\partial T} &= -(1/T^2)\left(\frac{\partial \tilde{u}}{\partial P} - (P/\rho^2)\frac{\partial \tilde{\rho}}{\partial P}\right), \\ (T/\rho^2)\frac{\partial \rho}{\partial T} &= \frac{\partial \tilde{u}}{\partial P} - (P/\rho^2)\frac{\partial \tilde{\rho}}{\partial P}, \\ \frac{\partial \tilde{u}}{\partial P} &= (T/\rho^2)\frac{\partial \rho}{\partial T} + (P/\rho^2)\frac{\partial \tilde{\rho}}{\partial P}. \end{aligned}$$

The last equation is (3.209).