Page 109, equation (3.148)

It follows from (3.62):

$$\mathrm{grad}\frac{\partial\rho}{\partial t} + \mathrm{grad}(\mathrm{div}\rho\mathbf{v}) = 0. \tag{1}$$

Capitalizing on (3.126) we obtain:

$$\mathrm{grad}\frac{\partial\rho}{\partial t} = \frac{\partial\mathbf{h}}{\partial t}. \tag{2}$$

The second term on the left hand side of (1) can be transformed as follows from its the component form:

$$\begin{aligned}\mathrm{grad}(\mathrm{div}\rho\mathbf{v}) &= \mathrm{grad}\left(\frac{\partial\rho v^1}{\partial x^1} + \frac{\partial\rho v^2}{\partial x^2} + \frac{\partial\rho v^3}{\partial x^3}\right)\\ &= \mathrm{grad}\left(v^1\frac{\partial\rho}{\partial x^1} + \rho\frac{\partial v^1}{\partial x^1} + v^2\frac{\partial\rho}{\partial x^2} + \rho\frac{\partial v^2}{\partial x^2} + v^3\frac{\partial\rho}{\partial x^3} + \rho\frac{\partial v^3}{\partial x^3}\right)\\ &= \mathrm{grad}(\mathbf{v}.\mathbf{h} + \rho\,\mathrm{div}\mathbf{v}) \equiv \mathrm{grad}(\mathbf{v}.\mathbf{h}) + \mathrm{grad}(\rho\,\mathrm{tr}\mathbf{D}) \qquad (3)\end{aligned}$$

where (3.16) was used in the last identity. The two final terms in (3) are further modified, one-by-one:

$$\mathrm{grad}(\rho\,\mathrm{tr}\mathbf{D}) = \rho\,\mathrm{grad}(\mathrm{tr}\mathbf{D}) + (\mathrm{tr}\mathbf{D})\,\mathrm{grad}\rho \equiv \mathbf{h}\,\mathrm{tr}\mathbf{D} + \rho\,\mathrm{grad}(\mathrm{tr}\mathbf{D}) \tag{4}$$

where (3.126) was used, again;

$$\begin{aligned}[\mathrm{grad}(\mathbf{v}.\mathbf{h})]^i &= \frac{\partial}{\partial x^i}(\mathbf{v}.\mathbf{h}) = v^1\frac{\partial h^1}{\partial x^i} + h^1\frac{\partial v^1}{\partial x^i} + v^2\frac{\partial h^2}{\partial x^i} + h^2\frac{\partial v^2}{\partial x^i} + v^3\frac{\partial h^3}{\partial x^i} + h^3\frac{\partial v^3}{\partial x^i}\\ &= v^1\frac{\partial h^i}{\partial x^1} + v^2\frac{\partial h^i}{\partial x^2} + v^3\frac{\partial h^i}{\partial x^3} + h^1L^{1i} + h^2L^{2i} + h^3L^{3i}. \qquad (5)\end{aligned}$$

In (5) definition (3.14) was used as well as the symmetry of the tensor grad $\mathbf{h}$ which is shown as follows:

$$(\mathrm{grad}\,\mathbf{h})^{ij} = \frac{\partial h^i}{\partial x^j} = \frac{\partial}{\partial x^j}\left(\frac{\partial\rho}{\partial x^i}\right) = \frac{\partial^2\rho}{\partial x^j\partial x^i} = \frac{\partial}{\partial x^i}\left(\frac{\partial\rho}{\partial x^j}\right) = \frac{\partial h^j}{\partial x^i} = (\mathrm{grad}\,\mathbf{h})^{ji}.$$

From (3.9) it follows:

$$\dot{h}^i = \frac{\partial h^i}{\partial x^j} + v^1\frac{\partial h^i}{\partial x^1} + v^2\frac{\partial h^i}{\partial x^2} + v^3\frac{\partial h^i}{\partial x^3}$$

and after substitution into (5):

$$[\mathrm{grad}(\mathbf{v}.\mathbf{h})]^i = \dot{h}^i - \frac{\partial h^i}{\partial t} + h^1L^{1i} + h^2L^{2i} + h^3L^{3i}$$

$$\mathrm{grad}(\mathbf{v}.\mathbf{h}) = \dot{\mathbf{h}} - \frac{\partial \mathbf{h}}{\partial t} + \mathbf{hL} \equiv \dot{\mathbf{h}} - \frac{\partial \mathbf{h}}{\partial t} + \mathbf{h}(\mathbf{D} + \mathbf{W}) \tag{6}$$

where (3.15) was used in the last identity; let us note that $\mathbf{h}(\mathbf{D}+\mathbf{W})$ denotes vector the i-th component of which is equal to $\sum_j h^j(D^{ji} + W^{ji})$.

Introducing (2), (3), (4), and (6) into (1), eq. (3.148) follows.