

Page 98, equation (3.113)

From equation (3.107) follows:

$$Q = \rho \dot{u} + \operatorname{div} \mathbf{q} - \operatorname{tr}(\mathbf{TD})$$

and equation (3.109) can be then modified:

$$-T\sigma = -T\rho\dot{s} - T \operatorname{div}(\mathbf{q}/T) + \rho\dot{u} + \operatorname{div} \mathbf{q} - \operatorname{tr}(\mathbf{TD})$$

From the definition (3.111) it follows that $\dot{f} = \dot{u} - T\dot{s} - s\dot{T}$; thus

$$-T\rho\dot{s} + \rho\dot{u} = \rho\dot{f} + \rho s\dot{T} \quad (1)$$

Further:

$$\begin{aligned} \operatorname{div}(\mathbf{q}/T) &= \sum_j \frac{\partial}{\partial x^j} \left(\frac{q^j}{T} \right) = \sum_j \left[\frac{1}{T} \frac{\partial q^j}{\partial x^j} + q^j \frac{\partial}{\partial x^j} \left(\frac{1}{T} \right) \right] \\ &= \frac{1}{T} \sum_j \frac{\partial q^j}{\partial x^j} + \sum_j q^j \left(\operatorname{grad} \frac{1}{T} \right)^j = \frac{1}{T} \operatorname{div} \mathbf{q} + \mathbf{q} \cdot \operatorname{grad} \frac{1}{T} \end{aligned} \quad (2)$$

and

$$\left(\operatorname{grad} \frac{1}{T} \right)^i = \frac{\partial}{\partial x^i} \left(\frac{1}{T} \right) = -\frac{1}{T^2} \frac{\partial T}{\partial x^i} = -\frac{1}{T^2} (\operatorname{grad} T)^i \equiv -\frac{1}{T^2} g^i \quad (3)$$

where the definition (3.112) was used in the last identity.

Taking into account (1) to (3) we obtain:

$$\begin{aligned} -T\sigma &= \rho\dot{f} + \rho s\dot{T} - T \left(\frac{1}{T} \operatorname{div} \mathbf{q} - \frac{1}{T^2} \mathbf{q} \cdot \mathbf{g} \right) + \operatorname{div} \mathbf{q} - \operatorname{tr}(\mathbf{TD}) \\ &= \rho\dot{f} + \rho s\dot{T} + T^{-1} \mathbf{q} \cdot \mathbf{g} - \operatorname{tr}(\mathbf{TD}) \end{aligned}$$

which is as in (3.113).