**Exercise 3 to section 4.7.**[1]

Apply the Svedberg formula on ultracentrifuge separation of uranium isotopes contained in $UF_6$. Suppose that $UF_6$ behaves as ideal gas (it sublimes at 50.5 °C) and contains only $U^{235}$ and $U^{238}$. Calculate the equilibrium content of $U^{235}$ (molar %) at the center of ultracentrifuge if its content at the ultracentrifuge perimeter equals to the natural proportion 0.72 % (molar). Ultracentrifuge diameter is 10 cm, speed 90 000 rpm, separation temperature is 60 °C.

Try to answer before continuing reading.

The formula, given on page 214, Rem. 23, is in this case:

$$(1/x_1)\,\mathrm{grad} x_1 = (M_1/RT)(1-\rho v_1)\omega^2 \mathbf{r}$$

where $x_1$ is the molar fraction of component 1 ($U^{235}F_6$), $M_1$ is its molar mass, $\rho$ is the mixture density and $v_1$ is the specific volume of pure component 1, $\omega$ is the angular velocity. The separation is limited along the axis perpendicular to the axis of rotation and the formula is simplified to one-dimensional form:

$$(1/x_1)\,(dx_1/dx) = (M_1/RT)(1-\rho v_1)\omega^2 x \tag{1}$$

where $x$ is the coordinate of the separation axis. The solution can be found in exercise 1 or 2 to section 4.7. Integrating (1) gives

$$\left[ -\frac{1}{1-\frac{M_2}{M_1}} \ln \frac{-(1-\frac{M_2}{M_1})x_1 + 1 - \frac{M_2}{M_1}}{x_1} \right]_{x_{01}}^{0.0072} = \frac{M_1\omega^2}{RT} \left[ \frac{x^2}{2} \right]_0^{0.05}. \tag{2}$$

The components are numbered 1=$U^{235}F_6$, 2=$U^{238}F_6$.

Solving (2) we obtain $x_{01} = 0.00813$. Thus the $U^{235}$ content which had to be calculated is 0.81 molar % (in the form of $U^{235}F_6$).

[1] Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).