Exercise 1 to section 4.7.[1]

Calculate the content of carbon dioxide (molar %) on the top of a gasometer of 50 m height. It operates at 25 °C and contains a mixture of carbon dioxide and hydrogen with equimolar composition at its bottom. The gases are ideal and the gravitational acceleration (g) has the value of 9.81 m s^{-2}.

Try to answer before continuing reading.

Hint: use the Svedberg formula given on page 214, Rem. 23.

The formula is:

$$(1/x_1)\,\mathrm{grad} x_1 = (M_1/RT)(1-\rho v_1)\mathbf{g}$$

where x_1 is the molar fraction of component 1 ($\mathrm{CO_2}$), M_1 is its molar mass, ρ is the mixture density and v_1 is the specific volume of pure component 1.

The gravitational acceleration has only one non-zero component (of magnitude g) which we locate along the vertical axis (symbol z). Thus, this axis points from top (of the gasometer) down; the gasometer top is located at $z = 0$ and its bottom at $z = 50\,m$. The Svedberg formula for this case is

$$(1/x_1)\,(dx_1/dz) = (M_1/RT)(1-\rho v_1)g. \tag{1}$$

The density of this two-component mixture is expressed using equations (4.419) and (4.424) from which it follows that:

$$\rho_\alpha = (P_\alpha M_\alpha)/RT = (x_\alpha P M_\alpha)/RT, \quad \alpha = 1,2 \tag{2}$$

and (1=$\mathrm{CO_2}$)

$$\rho = (P/RT)(x_{\mathrm{CO_2}} M_{CO_2} + x_{\mathrm{H_2}} M_{H_2}). \tag{3}$$

The product in (1) then is, taking into account also eq. (4.420),

$$\rho v_{\mathrm{CO_2}} = x_{\mathrm{CO_2}} + x_{\mathrm{H_2}}(M_{H_2}/M_{CO_2}) \tag{4}$$

or

$$\rho v_{\mathrm{CO_2}} = x_{\mathrm{CO_2}}\Big(1 - \frac{M_{H_2}}{M_{CO_2}}\Big) + \frac{M_{H_2}}{M_{CO_2}}. \tag{5}$$

Expression (5) is substituted into the Svedberg formula (1) and after a minor modification we have

$$\frac{dx_{\mathrm{CO_2}}}{x_{\mathrm{CO_2}}\Big[-x_{\mathrm{CO_2}}\Big(1-\frac{M_{H_2}}{M_{CO_2}}\Big) + 1 - \frac{M_{H_2}}{M_{CO_2}}\Big]} = \frac{gM_{CO_2}}{RT}dz. \tag{6}$$

[1]Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).

Eq. (6) is integrated using the limits on the left hand side from 0.5 to x (to be calculated), on the right hand side from 50 to 0 (meters), and $M_{CO_2} = 0.044$, $M_{H_2} = 0.002$ (kg/mol). The result is an equation for x the solution of which is $x = 0.4979$. Thus the carbon dioxide content at the bottom is 49.79% (molar).