Exercise 5 to section 3.7.[1]

Consider stationary heat conduction in plastic slab: area $114\,\mathrm{cm}^2$, thickness 0.64 cm, temperature of main surfaces 24 and 26 °C. Calculate the heat conductivity of the slab material if the heat conduction rate is 3 W and equations of exercise 4 to sec. 3.7 are valid.

Try to answer before continuing reading.

The stationary case means that $\partial T/\partial t = 0$ which is, according to equation (see exercise 1 to sec. 3.7)

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, \tag{1}$$

equivalent to $\partial^2 T/\partial x^2 = 0$.

We can calculate any of the two vanishing derivatives from the solution to (1) given in the exercise 4 to sec. 3.7. The result for the time derivative is:

$$\frac{\partial T}{\partial t} = \frac{Q}{4\rho c_V A\,\sqrt{\pi\alpha}} \exp\Big(\frac{-x^2}{4\alpha t}\Big) \frac{1}{\sqrt{t^3}} \Big(\frac{x^2}{2\alpha t} - 1\Big). \tag{2}$$

From (2) it is clear that stationary heat conduction occurs when

$$\frac{x^2}{2\alpha t} = 1$$

or

$$\alpha = \frac{x^2}{2t}. \tag{3}$$

From (3) and the definition of the temperature conductivity[2] it also follows that

$$\rho c_V = \frac{k}{\alpha} = \frac{2kt}{x^2}. \tag{4}$$

Substituting from (3) and (4) into the general solution of the heat equation given in exercise 4 to sec. 3.7

$$T(x,t) = T_o + \frac{Q}{\rho c_V A\, 2\sqrt{\pi\alpha t}} \exp\Big(\frac{-x^2}{4\alpha t}\Big) \tag{5}$$

we have for the slab main surface temperatures T_1 and T_2 ($T_2 > T_1$)

$$T_2 - T_1 = \frac{\dot{Q}\,x}{kA\sqrt{8\pi}} \exp(-1/2) \tag{6}$$

[1]Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).

[2]See exercise 1 to sec. 3.7.

where $\dot{Q} \equiv Q/t$ is the heat conduction rate.

Expressing k from (6), calculating with the given data, the value $k = 0.102$ J/(m s K) follows.