Exercise 4 to section 3.7.[1]

Verify that

$$T(x,t) = T_o + \frac{Q}{\rho c_V A\, 2\sqrt{\pi \alpha t}} \exp\left(\frac{-x^2}{4\alpha t}\right) \tag{1}$$

is a solution to the heat conduction equation (cf. exercise 1 to section 3.7)

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

($\alpha = k/(\rho c_V)$). Equation (1) describes uniaxial heat conduction along a rod with the cross-section A; T_o is the rod (and environment) temperature before the heat conduction was initiated by the heat Q at the point $x = 0$. This equation gives the temperature field in the rod.

Calculate the temperature field (in °C) in an ebonite rod for which $k = 0.167$ J/(m s K), $\rho = 1.15\,\mathrm{g/cm^3}$, and $c_V = 1.46\,\mathrm{J/(K\,g)}$, with $A = 1\,\mathrm{cm}^2$ and $T_o = 273.15\,\mathrm{K}$, by filling in following table:

	time (min)		
distance (mm)	1	2	5
0			
5			
10			

The answer is:

	time (min)		
distance (mm)	1	2	5
0	68.8	48.6	30.8
5	24.1	28.8	25.0
10	1.04	5.99	13.3

[1] Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).