Exercise 3 to section 3.8[1]

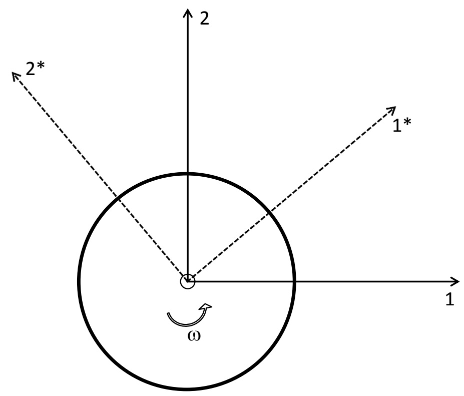
Derive the (differential) formula for the change of the ideal gas pressure with the distance from the rotor axis in a centrifuge under equilibrium (in the centrifugal field).

Calculate the equilibrium pressure of ideal gas at the outer edge of a centrifuge with the diameter of 10 cm, filled with nitrogen at 25 °C, rotating at 90 000 rev/min; the pressure at the rotor axis is 100 kPa.

Try to answer before continuing reading.

The equilibrium condition is given by (3.228). Body forces can be neglected (**b**=**o**). Acceleration in the frame connected with the rotor is given by (3.47).

The origin of the coordinate axis Nr. 1* is located in the rotor center and rotates with the centrifuge (rotor; the no-asterisk frame is fixed). We are thus interested (only) in the coordinate corresponding to this axis; let us denote this coordinate without the axis number (and asterisk) simply as x. See also the figure.



Then the relevant acceleration component i^1 is given by what follows from (3.47):

$$i^1 = \omega^2 x.$$

[1]Based on I. Samohýl: Irreversible Thermodynamics. Prague: University of Chemical Technology, 1998 (*in Czech*).

Inserting it into the condition (3.228) and using the ideal gas state equation we have

$$\frac{\partial P}{\partial x} \equiv \frac{\mathrm{d}P}{\mathrm{d}x} = \rho\,\omega^2 x = \frac{PM}{RT}\,\omega^2 x.$$

Rearranging gives the final formula:

$$\frac{\mathrm{d}\ln P}{\mathrm{d}x} = \frac{M\omega^2}{RT}\,x. \tag{1}$$

Answer to the numerical part. After integrating (1) from the center (where the pressure is P_0) to the position x we obtain the necessary formula:

$$\ln\frac{P}{P_0} = \frac{M\omega^2}{2RT}\,x.$$

Result: 350.5 kPa.