Page 70, equation (3.14)

Definition (3.7) shows that the velocity is defined on the basis of the function (3.1) which is a function of the type (3.4). Equation (3.2) shows that the variable $\mathbf{X}$ occurring in these functions can be expressed as a function of $\mathbf{x}$ (and of t). The gradient "grad" is defined in term of derivative with respect to $\mathbf{x}$, which can be in the case of the function (3.4) expressed using the chain rule. All these considerations can be put into symbols as follows.

$$\mathrm{grad}\mathbf{v} = \frac{\partial \mathbf{v}}{\partial \mathbf{x}} = \frac{\partial \mathbf{v}}{\partial \mathbf{X}}\frac{\partial \mathbf{X}}{\partial \mathbf{x}} = \frac{\partial \mathbf{v}}{\partial \mathbf{X}}\frac{\partial \underline{\chi}^{-1}(\mathbf{x},t)}{\partial \mathbf{x}} \equiv \frac{\partial \mathbf{v}}{\partial \mathbf{X}}\mathbf{F}^{-1} = \frac{\partial^2 \underline{\chi}(\mathbf{X},t)}{\partial \mathbf{X}\partial t}\mathbf{F}^{-1} = \frac{\partial}{\partial t}\left(\frac{\partial \underline{\chi}(\mathbf{X},t)}{\partial \mathbf{X}}\right)\mathbf{F}^{-1} \equiv \dot{\mathbf{F}}\mathbf{F}^{-1}.$$