## Page 149, Rem. 3, equation (*b*)

Equation (4.16) is localized using Gauss' theorem to give:

$$\sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha}{\partial t} + \sum_{\alpha=1}^{n} \mathrm{div} \rho_\alpha \mathbf{v}_\alpha = 0. \tag{1}$$

The time derivative follows from the definition (4.21):

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} \sum_{\alpha=1}^{n} \rho_\alpha = \sum_{\alpha=1}^{n} \frac{\partial \rho_\alpha}{\partial t}. \tag{2}$$

Further

$$\mathrm{div} \rho_\alpha \mathbf{v}_\alpha = \rho_\alpha \mathrm{div} \mathbf{v}_\alpha + \mathbf{v}_\alpha . \mathrm{grad} \rho_\alpha. \tag{3}$$

Using (2) and (3), Eq. (1) is rewritten:

$$\frac{\partial \rho}{\partial t} + \sum_{\alpha=1}^{n} \rho_\alpha \mathrm{div} \mathbf{v}_\alpha + \sum_{\alpha=1}^{n} \mathbf{v}_\alpha . \mathrm{grad} \rho_\alpha = 0. \tag{4}$$

Note that the second term in Eq. (4) appears also in the divergence of the barycentric velocity defined by Eq. (*a*) in Rem. 3 on page 149:

$$\mathrm{div} \mathbf{v}^w = \mathrm{div} \sum_{\alpha=1}^{n} w_\alpha \mathbf{v}_\alpha = (1/\rho) \sum_{\alpha=1}^{n} \rho_\alpha \mathrm{div} \mathbf{v}_\alpha + \sum_{\alpha=1}^{n} \mathbf{v}_\alpha . \mathrm{grad} w_\alpha. \tag{5}$$

An expression resembling the third term in Eq. (4) appears in the following scalar product involving the barycentric velocity:

$$\mathbf{v}^w . \mathrm{grad} \rho = \sum_{\alpha=1}^{n} w_\alpha \mathbf{v}_\alpha . \mathrm{grad} \rho. \tag{6}$$

The first term in (5) can be trivially modified as follows:

$$\rho (1/\rho) \sum_{\alpha=1}^{n} \rho_\alpha \mathrm{div} \mathbf{v}_\alpha = \sum_{\alpha=1}^{n} \rho_\alpha \mathrm{div} \mathbf{v}_\alpha. \tag{7}$$

The second term in (5) contains gradient of mass fraction; from its definition (4.22) follows that $\mathrm{grad}(\rho w_\alpha) = \mathrm{grad} \rho_\alpha$ and thus:

$$\rho \, \mathrm{grad} w_\alpha + w_\alpha \mathrm{grad} \rho = \mathrm{grad} \rho_\alpha. \tag{8}$$

From (8) it follows that

$$\text{grad}w_\alpha = (1/\rho)\text{grad}\rho_\alpha - (w_\alpha/\rho)\text{grad}\rho. \tag{9}$$

Substitution from (7) and (9) into (5) multiplied by the mixture density gives:

$$\begin{aligned}\rho\,\text{div}\mathbf{v}^w &= \sum_{\alpha=1}^{n}\rho_\alpha\text{div}\mathbf{v}_\alpha + \rho\sum_{\alpha=1}^{n}\mathbf{v}_\alpha.[(1/\rho)\text{grad}\rho_\alpha - (w_\alpha/\rho)\text{grad}\rho] \\ &= \sum_{\alpha=1}^{n}\rho_\alpha\text{div}\mathbf{v}_\alpha + \sum_{\alpha=1}^{n}\mathbf{v}_\alpha.\text{grad}\rho_\alpha - \sum_{\alpha=1}^{n}w_\alpha\mathbf{v}_\alpha.\text{grad}\rho. \end{aligned} \tag{10}$$

Summing up (10) and (6) gives:

$$\rho\,\text{div}\mathbf{v}^w + \mathbf{v}^w.\text{grad}\rho = \sum_{\alpha=1}^{n}\rho_\alpha\text{div}\mathbf{v}_\alpha + \sum_{\alpha=1}^{n}\mathbf{v}_\alpha.\text{grad}\rho_\alpha \tag{11}$$

Using (11) and definition ($c$) in Rem. 3 on page 149, Eq. (4) can be rewritten:

$$\frac{\partial\rho}{\partial t} + \mathbf{v}^w.\text{grad}\rho + \rho\,\text{div}\mathbf{v}^w \equiv \dot{\rho} + \rho\,\text{div}\mathbf{v}^w = 0. \tag{12}$$

Eq. (12) gives Eq. ($b$) in the same Rem.