Page 105, equation (3.132)

From (3.131) it follows:

$$\dot{f} = \frac{\partial \bar{f}}{\partial \mathbf{F}}.\dot{\mathbf{F}} + \frac{\partial \bar{f}}{\partial T}\dot{T} + \frac{\partial \bar{f}}{\partial \mathbf{g}}.\dot{\mathbf{g}} \tag{1}$$

From (3.14) we have $\dot{\mathbf{F}} = \mathbf{LF}$ and the first term on right hand side in (1) can be expressed taking into account also (3.15):

$$\begin{aligned}\frac{\partial \bar{f}}{\partial \mathbf{F}}.\mathbf{LF} =& \frac{\partial \bar{f}}{\partial \mathbf{F}}.(\mathbf{D}+\mathbf{W})\mathbf{F} = \frac{\partial \bar{f}}{\partial \mathbf{F}}.(\mathbf{DF}) + \frac{\partial \bar{f}}{\partial \mathbf{F}}.(\mathbf{WF}) = \mathrm{tr}\frac{\partial \bar{f}}{\partial \mathbf{F}}(\mathbf{DF})^T + \\ & \mathrm{tr}\frac{\partial \bar{f}}{\partial \mathbf{F}}(\mathbf{WF})^T = \mathrm{tr}\frac{\partial \bar{f}}{\partial \mathbf{F}}\mathbf{F}^T\mathbf{D} - \mathrm{tr}\frac{\partial \bar{f}}{\partial \mathbf{F}}\mathbf{F}^T\mathbf{W}. \end{aligned} \tag{2}$$

Note that also the definition of the scalar product of tensors was used: $\mathbf{A}.\mathbf{B} \equiv \mathrm{tr}\mathbf{AB}^T$, as well as the fact that $\mathbf{D}$ is symmetric whereas $\mathbf{W}$ anti-symmetric tensor, i.e., $\mathbf{D} = \mathbf{D}^T$ and $\mathbf{W} = -\mathbf{W}^T$.

The time derivative of temperature gradient can be expressed through the deformation gradient as indicated by (3.13), i.e., by $\mathbf{g} \equiv \mathrm{grad}\, T = (\mathrm{Grad}\, T)\mathbf{F}^{-1}$:

$$\dot{\overline{\mathrm{grad} T}} = (\dot{\overline{\mathrm{Grad} T}})\,\mathbf{F}^{-1} + (\mathrm{Grad} T)\,\dot{\overline{\mathbf{F}^{-1}}} \tag{3}$$

As stated in the book, $\dot{\overline{\mathbf{FF}^{-1}}} = 0$ (because this is time derivative of unit, constant, tensor), from which we get:

$$\dot{\overline{\mathbf{FF}^{-1}}} = \dot{\mathbf{F}}\mathbf{F}^{-1} + \mathbf{F}\dot{\overline{\mathbf{F}^{-1}}} = 0 \quad \Rightarrow \quad \dot{\overline{\mathbf{F}^{-1}}} = -\mathbf{F}^{-1}\dot{\mathbf{F}}\mathbf{F}^{-1}. \tag{4}$$

Substituting from (2)-(4) into (1) results in:

$$\begin{aligned}\dot{f} =& \mathrm{tr}\frac{\partial \bar{f}}{\partial \mathbf{F}}\mathbf{F}^T\mathbf{D} - \mathrm{tr}\frac{\partial \bar{f}}{\partial \mathbf{F}}\mathbf{F}^T\mathbf{W} + \frac{\partial \bar{f}}{\partial T}\dot{T} + \\ & \frac{\partial \bar{f}}{\partial \mathbf{g}}.\left[(\dot{\overline{\mathrm{Grad} T}})\,\mathbf{F}^{-1} - (\mathrm{Grad} T)\,(\mathbf{F}^{-1}\dot{\mathbf{F}}\mathbf{F}^{-1})\right]. \end{aligned} \tag{5}$$

The two terms in square brackets can be further modified (after multiplication by $\partial \bar{f}/\partial \mathbf{g}$). This procedure will be illustrated on the first term only which is of the type $\mathbf{a}.(\mathbf{bC})$; in the component form:

$$\begin{aligned}\mathbf{a}.(\mathbf{bC}) =& \sum_i a^i(\mathbf{bC})^i = \sum_i a^i \sum_j b^j C^{ji} = \sum_i \sum_j C^{ji} a^i b^j \equiv \\ & \sum_i \sum_j C^{ij} a^j b^i = (\mathbf{Ca}).\mathbf{b} \end{aligned} \tag{6}$$

The modifications indicated in (6) result in following equalities:

$$\frac{\partial \bar{f}}{\partial \mathbf{g}} . \left[\left(\overline{\mathrm{Grad} T}^{\cdot} \right) \mathbf{F}^{-1} \right] = \left(\mathbf{F}^{-1} \frac{\partial \bar{f}}{\partial \mathbf{g}} \right) . \overline{\mathrm{Grad} T}^{\cdot} \tag{7}$$

$$\frac{\partial \bar{f}}{\partial \mathbf{g}} . \left[\left(\mathrm{Grad} T \right) \left(\mathbf{F}^{-1} \dot{\mathbf{F}} \mathbf{F}^{-1} \right) \right] = \left(\mathbf{F}^{-1} \frac{\partial \bar{f}}{\partial \mathbf{g}} \right) . \left[\left(\mathrm{Grad} T \right) \left(\mathbf{F}^{-1} \dot{\mathbf{F}} \right) \right] . \tag{8}$$

Substituting from (7) and (8) into (5) and then into (3.113) gives (3.132).